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Three-dimensional flow induced by the torsional motion of a cylinder

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Abstract

The flow induced outside a highly flexible cylindrical sheet executing pure torsional motion is studied. The problem is governed by the torsional Reynolds number \( R = \gamma a^2/\nu \), where \( \gamma \) is the axial rate of rotation, \( a \) is the cylinder radius and \( \nu \) is the fluid kinematic viscosity. An interesting feature of this problem is that the axial pressure gradient of the primary flow induces a weak transverse flow in the meridional plane. The axial component of this motion takes the form of a wall jet. The high Reynolds number asymptotics for the shear stress parameters and the radially entrained flow are presented and compared with full numerical results computed over the large range of Reynolds numbers \( 10^{-2} \leq R \leq 10^6 \).

1. Introduction

This paper continues an ongoing study of fluid motion induced by stretching and shearing surfaces. After the seminal paper by Crane (1970) reporting the exact solution of the Navier–Stokes (NS) equations for the motion induced by a linearly stretching plate, a growing number of boundary-layer flow studies, with and without heat and mass transfer, have appeared in the literature. We note for historical accuracy that the solution oft-quoted as Crane (1970) was originally discovered by Riabouchinsky (1924); see Drazin and Riley (2006). Brady and Acrivos (1981) studied the exact NS flow inside a cylindrical tube driven by linear extensional boundary motion. Wang (1988) reported exact NS solutions for flow outside a linearly stretching cylinder. Marqués et al (1998) provided a generalized exact NS formulation for Couette–Poiseuille flow that includes both extensional and torsional motion of a cylinder surface; however, no torsional solutions were given. Weidman and

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Figure 1. Schematic illustration of the infinite-cylinder geometry. The velocity profile indicates the outer-wall velocity at $\theta = \pi/2$, which is given by $u = w = 0$ and $v = \gamma z$.

Magyari (2010) reported boundary-layer solutions induced by arbitrary rectilinear extensional motion of a planar surface that, in general, must be accompanied by a specific form of wall transpiration. Very recently, Weidman (2010) presented exact NS and Stokes flow solutions induced exterior/interior to or between concentric cylindrical sheets executing periodic surface stretching or shearing motions.

In this work, the pure stretching motion of a cylinder studied by Wang (1988) is replaced by a pure torsional motion. We naturally employ the cylindrical coordinates $(r, \theta, z)$ while denoting the respective coordinate velocities as $(u, v, w)$. In Wang’s problem, the flow outside the cylinder was driven by linear axial stretching $w = kz$ of the cylinder surface. In that case there is no external axial pressure gradient, the flow is axisymmetric, and there is no induced swirling motion; hence the flow is two dimensional in meridional planes. Now we consider the exterior flow driven by linear torsional motion $v = \gamma z$ of the cylindrical surface. Again there is no external axial pressure gradient and the flow is axisymmetric. However, the $z$-dependent swirling motion induces an axial pressure gradient in the neighborhood of the cylinder and thus the flow is fully three dimensional.

We note a closely related study by O’Dea and Waters (2006), who investigated solute uptake inside a twisting tube. Their interest was in the time-periodic motion of flexible tubes modeling coronary artery flow. The muscle fibers of the heart are so arranged as to result in a wringing type of motion that efficiently squeezes blood from the heart with each cardiac beat. In turn the vessels themselves are subject to a periodic wringing motion as they twist and untwist with each beat. O’Dea and Waters (2006) note that the time-periodic twisting motion of flexible vessels induces, at second order, a mean axial pressure gradient that is responsible for a steady-streaming axial flow along the artery. Thus O’Dea and Waters (2006) were the first to identify this mechanism for the generation of axial flow in flexible tubes. In this work, we show that time-periodic torsional motion is not the only means by which an axial pressure gradient may be generated. Indeed, a steady torsional motion of a tube also induces a steady axial pressure gradient. In this work, however, we consider the flow induced exterior to the cylinder.

This paper is organized as follows. In section 2, the similarity reduction of the NS equations is presented. Leading low Reynolds number behavior is given in section 3, high Reynolds number asymptotics are developed in section 4, and numerical solutions are presented in section 5. A discussion of results and the concluding remarks are given in section 6.

2. Problem formulation

The cylinder has radius $a$ and the flow exterior to the cylinder is driven by the linear torsional motion $v = \gamma z$ of its surface; see figure 1. For the steady axisymmetric flow under
consideration, the continuity and NS equations in cylindrical coordinates are

\[(ru)_r + (rw)_z = 0, \quad (2.1a)\]
\[uu_r + uu_z - r^{-1}v^2 = -\rho^{-1}p_r + \nu(u_{rr} + r^{-1}u_r + u_{zz} - r^{-2}u), \quad (2.1b)\]
\[uv_r + uv_z + r^{-1}uv = \nu(v_{rr} + r^{-1}v_r + v_{zz} - r^{-2}v), \quad (2.1c)\]
\[uw_r + uw_z = -\rho^{-1}p_r + \nu(w_{rr} + r^{-1}w_r + w_{zz}), \quad (2.1d)\]

where \(\rho\) is the fluid density, \(p\) is the pressure and \(v\) is the kinematic viscosity of the fluid. The posited solution ansatz for the three-dimensional flow represents an adaptation and slight modification of that used, in a related problem, by Cunning et al (1998),

\[u = -U_0 f(\eta) \left(\frac{z}{a}\right), \quad v = V_0 g(\eta) \left(\frac{z}{a}\right), \quad w = 2U_0 f'(\eta) \left(\frac{z}{a}\right), \quad p = \rho U_0^2 P(\eta, z), \quad (2.2)\]

where \(V_0 = \gamma a\) is a reference torsional velocity, \(U_0 = v/a\) is a reference diffusion velocity and the prime denotes differentiation with respect to \(\eta = (r/a)^2\). The continuity equation \((2.1a)\) is satisfied identically and the three momentum equations become

\[P_\eta = \frac{f^2}{2\eta^2} - \frac{f f'}{\eta} - 2f'' + R^2 \frac{g^2}{2\eta} \left(\frac{z}{a}\right)^2, \quad (2.3a)\]
\[\eta g'' + g' - \frac{g}{4\eta} = \frac{1}{2} (f' g - fg') - \frac{fg}{4\eta}, \quad (2.3b)\]
\[P_z = \left(\frac{f}{a}\right) \left[2(\eta f'' + f'') + ff'' - f^2\right] \left(\frac{z}{a}\right), \quad (2.3c)\]

Equating cross-differentiations of \((2.3a)\) and \((2.3c)\) furnishes, with the addition of \((2.3b)\), the governing equations of motion

\[2(\eta f'' + f'') + ff'' - f^2 = R^2 \frac{g^2}{4\eta}, \quad (2.4a)\]
\[\eta g'' + g' - \frac{g}{4\eta} + \frac{1}{2} (fg' - f'g) + \frac{fg}{4\eta} = 0. \quad (2.4b)\]

The cylinder wall is impermeable and its azimuthal velocity is \(\gamma z\). Far from the cylinder the induced axial and azimuthal velocities tend to zero, as does \(P_\eta\) in equation \((2.3a)\). These give rise to the associated boundary conditions

\[f(1) = 0, \quad f'(1) = 0, \quad f'(\infty) = 0, \quad f''(\infty) = 0, \quad (2.5a)\]
\[g(1) = 1, \quad g(\infty) = 0, \quad (2.5b)\]

in which the sole governing parameter is the torsional Reynolds number

\[R = \frac{\gamma a^2 \nu}{\nu}. \quad (2.6)\]

The compatible pressure field associated with the flow obtained by integration of equations \((2.3a)\) and \((2.3c)\) is

\[P(\eta, z) = P_0 - \frac{1}{2} \left[\frac{f^2}{\eta} + 4f' - \left(\frac{z}{a}\right)^2 \int_1^{\xi} \frac{g(\xi)^2}{\xi} d\xi\right]. \quad (2.7)\]
Of interest is the wall shear stress generated by the induced fluid motion. The wall stresses in the axial and azimuthal directions are given by

\[
\tau_{rz,\text{wall}} = \mu \left. \frac{\partial w}{\partial r} \right|_{r=a} = \frac{4\rho v^2}{a^2} \frac{f''(1)}{a} \left( \frac{z}{a} \right),
\]

(2.8a)

\[
\tau_{r\theta,\text{wall}} = \mu \left[ \frac{\partial v}{\partial r} - \frac{v}{r} \right] = \rho \nu \gamma \left[ 2g'(1) - 1 \right] \left( \frac{z}{a} \right),
\]

(2.8b)

where \( \mu \) is the viscosity and \( f''(1) \) and \( g'(1) \) are shear stress parameters obtained from the solution of the boundary-value problem. The radially entrained velocity decays at infinity in the manner

\[
u \sim -\frac{v f(\infty)}{a} \frac{1}{\eta^{1/2}} \quad (\eta \to \infty),
\]

(2.9)

where \( f(\infty) \) denotes the radial entrainment parameter. Finally, we observe that the torsional motion of the cylinder produces radial vorticity \( \zeta \) threading the fluid. The value of that vorticity generated at the wall is

\[
\zeta_{\text{wall}} = -\left. \frac{\partial v}{\partial z} \right|_{r=a} = -\gamma.
\]

(2.10)

3. Small- \( R \) asymptotics

We exhibit the zero Reynolds number behavior as part of an effort to compute the low- \( R \) asymptotics. Assuming a regular expansion of the form

\[
f(\eta) = R f_0(\eta) + R^2 f_1(\eta) + \cdots,
\]

(3.1a)

\[
g(\eta) = g_0(\eta) + R g_1(\eta) + \cdots,
\]

(3.1b)

one obtains, upon insertion into equations (2.4), a series of problems to solve at each power of \( R \). The \( O(R) \) problem is

\[
[\eta f_0'' + f_0']' = 0, \quad f_0(1) = f_0'(1) = f_0''(\infty) = f_0''(\infty) = 0,
\]

(3.2a)

\[
\eta^2 g_0'' + \eta g_0' - \frac{1}{4} g_0 = 0, \quad g_0(1) = 1, \quad g_0(\infty) = 0.
\]

(3.2b)

The solution of this lowest-order system is simply

\[
f_0(\eta) = 0,
\]

(3.3a)

\[
g_0(\eta) = \frac{1}{\eta^{1/2}},
\]

(3.3b)

uniformly valid in \( \eta \). Continuation to next order shows the appearance of logarithmic terms and nonuniformity in \( \eta \). Thus the expansion (3.1) can be considered as the inner solution and it is clear that matching to an outer solution requires switchback to insert terms of the form \( \eta \ln \eta \), much like in the classic problem of low-Reynolds-number flow over a circular cylinder (Hapel and Brenner 1983). This tedious analysis will not be pursued here.

However, we have learned that the wall stress parameter \( f''(1) \) of the induced meridional flow is \( o(R) \) and the leading behavior of the wall stress parameter corresponding to the primary azimuthal flow is

\[
g'(1) \sim -\frac{1}{2},
\]

(3.4a)
which gives the low-

\[ \tau_{w} \sim -2 \rho v \gamma \left( \frac{z}{\alpha} \right). \]  

\[ (3.4b) \]

4. Large-\( R \) asymptotics

Introducing the small parameter \( \epsilon = 1/R \) and the change of variables

\[ f(\eta) = \frac{2}{\epsilon} F(\eta), \quad g(\eta) = 2G(\eta), \quad (4.1) \]

the governing equations (2.4) may be written as

\[ [\epsilon (\eta F''' + F' F' - F'') = \frac{1}{4} \frac{G^2}{\eta}, \quad (4.2a) \]

\[ \epsilon \left[ \eta G'' + G' - \frac{1}{4} \frac{G}{\eta} \right] + FG' - F' G + \frac{1}{2} \frac{FG}{\eta} = 0, \quad (4.2b) \]

to be solved with boundary conditions

\[ F(1) = 0, \quad F'(1) = 0, \quad F'(\infty) = 0, \quad F''(\infty) = 0, \quad (4.3a) \]

\[ G(1) = \frac{1}{2}, \quad G(\infty) = 0. \quad (4.3b) \]

Thus we have a singular perturbation problem that suggests the need for matching inner and outer solutions on both \( f(\eta) \) and \( g(\eta) \).

4.1. Outer region

In the outer region, we assume the regular perturbation expansions

\[ F(\eta) = F_0(\eta) + \epsilon F_1(\eta) + \cdots, \quad (4.4a) \]

\[ G(\eta) = G_0(\eta) + \epsilon G_1(\eta) + \cdots, \quad (4.4b) \]

and obtain the coupled leading-order equations

\[ [F_0 F_0'' - F_0'^2] = \frac{1}{4} \frac{G_0^2}{\eta}, \quad (4.5a) \]

\[ F_0 G_0' - F_0 G_0 + \frac{1}{2} \frac{F_0 G_0}{\eta} = 0, \quad (4.5b) \]

to be solved with \( F_0' \) and \( G_0 \) decaying in the far field. The solutions are readily found to be

\[ F_0(\eta) = A \eta e^{-\alpha \eta}, \quad G_0(\eta) = A \sqrt{8\alpha} \eta^{1/2} e^{-\alpha \eta}, \quad (4.6) \]

where the prefactor \( A \) and the exponent \( \alpha \) have not been determined.

4.2. Inner region

In the boundary-layer region, we introduce the stretched wall coordinate \( \xi \) and dependent variables

\[ \eta = 1 + \delta(\epsilon) \xi, \quad F(\eta) \to F(\xi), \quad G(\eta) \to G(\xi), \quad (4.7) \]
to obtain from equations (4.2) the coupled system

\[(1 + \delta \xi)[(1 + \delta \xi)\varepsilon F'' + \varepsilon \delta F'' + \delta(FF' - F^2)]' = \frac{\delta^4}{4} G^2, \tag{4.8a}\]

\[(1 + \delta \xi)^2 \varepsilon G'' + (1 + \delta \xi) \delta G'' - \frac{\delta^2}{4} G + (1 + \delta \xi) \delta (FG' - F'G) + \frac{\delta^2}{2} FG = 0, \tag{4.8b}\]

where now a prime denotes differentiation with respect to \( \xi \). Our numerical calculations show that \( F = o(G) \) with \( G = O(1) \). Thus the leading-order behaviors

\[F(\xi) = \mu(\varepsilon) F_0(\xi), \quad G(\xi) = G_0(\xi), \tag{4.9}\]

are anticipated, where \( \mu(\varepsilon) \) and \( \delta(\varepsilon) \) are yet to be determined. Equations (4.8) now yield

\[(1 + \delta \xi)[(1 + \delta \xi)\varepsilon F_0'' + \varepsilon \delta F_0'' + \delta \mu F_0 + \delta \mu^2 (F_0 F_0' - F_0'^2)]' = \frac{\delta^4}{4} G_0^2, \tag{4.10a}\]

\[(1 + \delta \xi)^2 \varepsilon G_0'' + (1 + \delta \xi) \delta G_0'' - \frac{\delta^2}{4} G_0 + (1 + \delta \xi) \delta \mu (F_0 G_0' - F_0' G_0) + \frac{\delta^2}{2} \mu F_0 G_0 = 0. \tag{4.10b}\]

The highest-order derivatives in \( F_0 \) and \( G_0 \) need to be retained for satisfying the wall boundary conditions. Furthermore, since the meridional flow is driven by the axial gradient of the pressure in (2.7), the term on the right-hand side of equation (4.10a) must be retained, giving the balance \( \varepsilon \mu = \delta^4 \). To match with the outer flow, at least one term in the outer-flow equation (4.5b) should be retained in equations (4.10b). Since both \( \delta(\varepsilon) \) and \( \mu(\varepsilon) \) tend to zero as \( \varepsilon \to 0 \), it is the term proportional to \( \delta \mu \) that is retained giving the balance \( \varepsilon = \delta \mu \). Consequently,

\[\delta(\varepsilon) = \varepsilon^{2/5}, \quad \mu(\varepsilon) = \varepsilon^{3/5}, \tag{4.11}\]

and hence the leading terms in equations (4.10) are

\[F_0'' + F_0 F_0'' - F_0' F_0'' = \frac{1}{4} G_0^2, \quad G_0'' + F_0 G_0' - F_0' G_0 = 0, \tag{4.12}\]

to be solved with wall and far-field conditions

\[F_0(0) = F_0'(0) = F_0'(\infty) = F_0''(\infty) = 0, \quad G_0(0) = \frac{1}{2}, \quad G_0(\infty) = 0. \tag{4.13}\]

In singular perturbation problems of this type, boundary conditions in the far field are generally not satisfied by the inner solution, and matching of the inner and outer solutions is required. In this case, however, we find that governing equations (4.12) can be solved satisfying both wall and far-field conditions (4.13). Integration of this system gives \( F_0''(0) = 0.0972949 \), \( G_0''(0) = -0.137384 \) and \( F_0(\infty) = 0.383352 \). Working back through the transformations, one obtains the leading-order behaviors of the shear stress parameters

\[g'(1) \sim 2 R^{2/5} G_0'(0), \tag{4.14a}\]

\[f''(1) \sim 2 R^{6/5} F_0''(0) \tag{4.14b}\]

and the radial entrainment parameter

\[f(\infty) \sim 2 R^{2/5} F_0(\infty). \tag{4.15}\]
5. Numerical results

Following Wang (1988) we employ the change of variable $\eta = e^x$, which effectively compresses the required domain for numerical solution. Under this transformation, the governing equations (2.4) may be written as

$$f_{xxxx} - 4 f_{xxx} + 5 f_{xx} - 2 f_x + \frac{1}{2} \left[ f \left( f_{xxx} - 3 f_{xx} + 2 f_x \right) - f_x \left( f_{xx} - f_x \right) \right] = \frac{1}{R^2} e^{2x} g^2, \quad (5.1a)$$

$$g_{xx} - \frac{1}{4} g + \frac{1}{2} \left( f g_x - f_x g \right) + \frac{f g}{4} = 0, \quad (5.1b)$$

subject to the boundary conditions

$$f(0) = 0, \quad f_x(0) = 0, \quad \lim_{x \to \infty} e^{-x} f_x = 0, \quad \lim_{x \to \infty} e^{-2x} (f_{xx} - f_x) = 0, \quad (5.2a)$$

$$g(0) = 1, \quad \lim_{x \to \infty} g = 0. \quad (5.2b)$$

Results were obtained with a Matlab® routine built on the *bvp4c* boundary-value-problem solver. For a given $R < 1$, $f(x)$ varied so slowly with $x$ that a small variation in the radial entrainment parameter $f(x_{\text{max}}) \approx f(\infty)$ was observed near the largest numerically obtainable values of $x_{\text{max}}$. Thus, the radial entrainment parameter $f(\infty)$ was determined by extrapolation of $f(x_{\text{max}})$ from the largest $x_{\text{max}}$ values.

Similarity profiles for the primary flow $g(\eta)$ are presented in figure 2 at selected Reynolds numbers in the range $0.01 \leq R \leq 1000$. Note that as $R \to 0$ these numerical solutions accurately approach the zero Reynolds number solution $g(\eta) = \eta^{-1/2}$ found in section 3; indeed the solutions shown for $R = 0.01$ and $R = 0$ in figure 2 are virtually indistinguishable.

Similarity profiles for the axial flow $f'(\eta)$, induced by the axial pressure gradient of the primary flow, are shown in figure 3 for selected values of $R$. The linear scale for $f'(\eta)$ in figure 3(a) shows clearly that $f'(1) = 0$ and that $f'(\eta)$ decays in the far field, at least for $R = 10, 100$ and 1000. To exhibit the small Reynolds number results we display $f'(\eta)$ using
a logarithmic scale in figure 3(b). Here it is clear that the boundary layer of the induced axial flow grows dramatically as $R$ decreases, and that the peak of the induced wall jet increases dramatically as $R$ increases. The induced axial flow is reminiscent of the classic wall jet originally found by Teterin (1948) and later independently rediscovered by Akatnov (1953) and Glaeuer (1956). To call attention to the less well-known works of Teterin and Akatnov, we denote this as the TAG wall jet; see Magyari and Weidman (2005). Continuity requires that our wall jet induces a radial inflow of the form $f(\eta)/\sqrt{\eta}$, which we display in figure 4 for selected values of $R$, again using both linear and logarithmic scales to best elucidate the large variation of solutions with Reynolds number.

Results of extensive numerical integrations give the primary, azimuthal wall shear stress parameter $g'(1)$ plotted in figure 5 over the range $10^{-2} \leq R \leq 10^6$. The small-$R$ behavior given by equation (3.4a) and the large-$R$ asymptotics given by equation (4.14a) are shown by the dashed lines. The results for the secondary, axial wall shear stress parameter $f''(1)$ are shown in figure 6 along with the large-$R$ asymptotics given by equation (4.14b) plotted as a dashed line. Finally, figure 7 shows the radial entrainment parameter $f(\infty)$ compared with its
large-\( R \) asymptotic behavior given by equation (4.15). In each of these parametric plots, the large-\( R \) asymptotics provide accurate representations for \( R \geq 1000 \).

6. Discussion and conclusion

The flow external to a circular cylinder in pure torsional motion is studied as an exact solution to the NS equations. The primary swirling motion produced by the torsion induces a secondary flow in the meridional plane, the axial component of which has the appearance of a wall jet. This jet, however, is distinctly different from the classical TAG jet in that there is no conserved exterior momentum flux, as found by Tetervin (1948), Akatnov (1953) and Glauert (1956) for the two-dimensional wall jet. Due to viscous diffusion, the TAG jet attenuates downstream, while the wall jet in the present problem increases in strength due to the axial pressure gradient proportional to \( z \) that drives the flow near the boundary.

The numerical results are obtained over a wide range of Reynolds numbers. This range \( 10^{-2} \leq R \leq 10^6 \) is sufficient to capture the high-\( R \) asymptotics both for the shear
stress parameters $g'(1)$ and $f''(1)$ and for the radial entrainment parameter $f(\infty)$. Indeed, the large $R$ numerics are found to be accurate representations of these parameters when $R \gtrsim 1000$.

The flow is likely to undergo instability at sufficiently large Reynolds numbers. It might be anticipated that the primary flow governs the stability of the fully three-dimensional flow, in which case the flow may become centrifugally unstable. To pursue this line of reasoning we compute the Rayleigh criterion (Drazin and Reid 2004) for inviscid centrifugal instability

\[
\frac{d\Gamma^2}{dr} < 0,
\]  

(6.1)
where $\Gamma = rv(r)$ is the circulation. Were the flow inviscid and composed only of the primary swirl component, we would have

$$\Gamma = rv_0 g(\eta) \left( \frac{z}{a} \right),$$

so that

$$\frac{d\Gamma^2}{dr} = \frac{2v_0^2}{a^2} \epsilon^2 \eta^{3/2} g[g + g']\eta.'$$

Since $0 \leq g(\eta) \leq 1$, Rayleigh’s criterion (6.1) reduces to

$$g + g' < 0$$

for instability. Inserting the Stokes flow solution $g(\eta) = \eta^{-1/2}$ into equation (6.4) gives $\eta < 1/2$, which is never achieved because $\eta \geq 1$. Hence, were the Stokes flow inviscid, it would be stable. However, since $g(\eta)$ is bounded by one and $g'(\eta)$ becomes increasingly negative with increasing $R$, there comes a value $R = 14.7$ at which the inviscid swirl flow first becomes centrifugally unstable. The actual stability of this three-dimensional flow remains an open problem for future study. However, we have fully determined the base flow required for any future stability calculation.

Two extensions of this work are under consideration. The first one is the fluid flow inside a cylinder undergoing pure torsional motion. Here we expect to see the steady induced axial pressure gradient drive the fluid down the cylinder, distinct from the steady streaming behavior studied by O’Dea and Waters (2006). At the next level of complication is the flow induced by the spirular motion of a cylinder, where both linear stretching and linear torsional motions are superposed. Both the stretching and torsional motion induce axial pressure gradients, which should enhance the axial motion along the cylinder. At low Reynolds number, this latter problem may have applications in the cytoplasmic streaming of certain species of plants; see, for example, van de Meent et al (2008).

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